Can One Divide Uncertainty into Two Kinds?

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Two kinds of uncertainty

Earthquake engineers and seismologists commonly try to distinguish between two categories of uncertainty: aleatory (having to do with inherent randomness) and epistemic (having to do with one's model of nature). Does the distinction reflect reality?

Figure 1. Left: a die (alea) literally symbolizes aleatory uncertainty. Right: Thomas Bayes, under whose eponymous viewpoint all uncertainty is epistemic (image credits: (L) Charles Rondeau, (R) unknown; both public domain)

Aleatory uncertainty is supposedly inherent

Aleatory uncertainties are supposedly irreducible, existing in nature because they are inherent—natural—to the process involved. The roll of dice (alea is a single die in Latin) or the toss of a coin are cited as examples of irreducible, inherent randomness. Their outcome probabilities are conceived as existing in nature, inherent in the process in question, and with infinite repeated trials the probabilities can be determined with certainty but not changed. An example of a possibly aleatory uncertainty from earthquake engineering is the uncertainty in structural response resulting from randomness in the ground motion, sometimes called the record-to-record variability.

Epistemic uncertainty is all in the mind

Epistemic uncertainties are supposedly reducible with better knowledge, such as with a better structural model or after more experimental testing of a component. They exist as attributes of the mathematical model, that is, because of the knowledge state of the modeler. They do not exist in nature. They are not inherent in the real-world process under consideration.

Most US earthquake engineers and seismologists at the time of this writing seem to hold this view of probability—that uncertainties can be classified as aleatory or epistemic—a view that one can call the frequentist or classical view.

But maybe there is only one kind

The frequentist viewpoint is not unchallenged, the alternative being so-called Bayesian probability. Beck (2009) advances the Bayesian viewpoint, arguing first that aleatory uncertainty is vaguely defined. More importantly, he points out that one cannot scientifically prove that any quantity is inherently uncertain, that better knowledge of its value cannot be acquired. Under this viewpoint, all uncertainty springs from imperfections in our model of the universe—all uncertainty is epistemic.

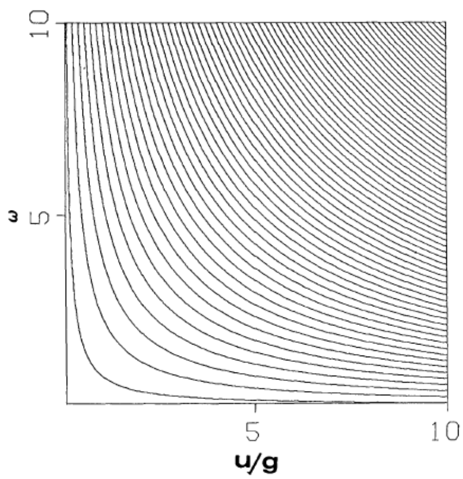
Can one reconcile the frequentist and Bayesian viewpoints?

Der Kiureghian and Ditlevsen (2009), who seem to be trying to square the circle and reconcile the frequentist and Bayesian viewpoints (note the title of their work: “Aleatory or epistemic? Does it matter?”), offer this definition: “Uncertainties are characterized as epistemic if the modeler sees a possibility to reduce them by gathering more data or by refining models. Uncertainties are categorized as aleatory if the modeler does not foresee the possibility of reducing them.” Under these pragmatic definitions, aleatory or epistemic depends on the knowledge state or belief of the modeler: an uncertainty is aleatory if the modeler thinks it cannot be practically reduced in the near term without great scientific advances and epistemic otherwise. Under this definition an uncertainty can be aleatory to one modeler and epistemic to another. The authors suggest that “these concepts only make unambiguous sense if they are defined within the confines of a model of analysis.” Which seems to mean that although these authors use the word aleatory, they mean something different than inherent randomness.

What if one can predict a coin toss?

Let us test the distinction by looking more closely at a favorite frequentist example: the coin toss. Suppose one tossed the coin over sand or mud, a surface from which the coin will not bounce, with initial elevation above the surface $y = 0$, initial upward velocity u and initial angular velocity ω, and initially heads up. The calculation of the coin-toss outcome becomes a problem of Newtonian mechanics, which does not acknowledge uncertainty. Keller (1986) offers the solution shown in Figure 3A. Diaconis et al. (2007) demonstrated deterministic coin-tossing with a laboratory experiment (see Figure 3B for their device), concluding that “coin-tossing is physics, not random.” Without the initial information, the process appears random (what subsequent authors called coarse-grained random); with the initial information it becomes fine-grained deterministic. The additional information eliminates the supposedly irreducible aleatory uncertainty.

Figure 2. Does a coin toss represent an irreducible uncertainty? (image credit: ICMA Photos, Attribution-ShareAlike 2.0 Generic license)



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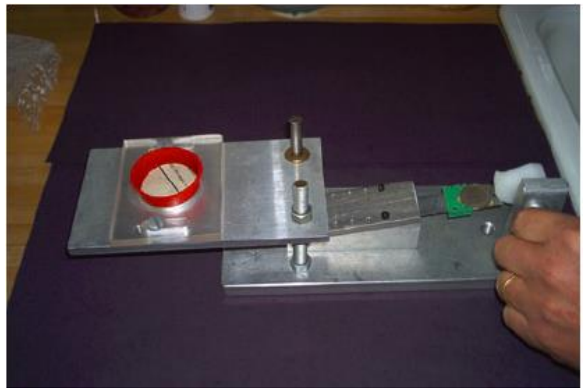
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Figure 3. A. Keller's (1986) curves separating coin-toss solutions for heads and tails for a coin tossed from elevation 0 with initial upward velocity u and angular velocity ω. B. Diaconis et al.'s (2007) coin-tossing device

Reducing an irreducible uncertainty in seismology

Let us consider another example from seismology: record-to-record uncertainty. Seismologists have created computational models of faults and the mechanical properties of the lithosphere and surficial geology, producing modeled ground motions for specified fault ruptures. See for example Graves and Somerville (2006) or Aagaard et al. (2010a, b). These seem likely to be more realistic than those drawn from a database of ground motions recorded from other sites with a variety of site conditions and seismic environments dissimilar from the sites of interest, again reducing the supposedly inherently and irreducibly random.

More knowledge sometimes increases uncertainty

Let us next consider the notion that epistemic uncertainties can be reduced with more knowledge. In fact, often new knowledge increases uncertainty rather than decreasing it. Our initial models may be drawn from too little data or data that do not reflect some of the possible states of nature. Or they may be based on overly confident expert judgment. For example, until about 2000, seismologists believed that a fault rupture could not jump from one fault to another. They have since observed such fault-to-fault ruptures, e.g., in the 2002 Denali Alaska Earthquake. The new knowledge led the seismologists to abandon the notion that the maximum magnitude of an earthquake was necessarily limited by the length of the largest fault segment. Their uncertainty as to the maximum magnitude of earthquakes elsewhere increased as a result, e.g., between the 2nd and 3rd versions of the Uniform California Earthquake Rupture Forecasts (Field et al. 2007, 2013).

How do frequentists, Bayesians, and middle-grounders view a horse race?

The viewpoints discussed here are held on the one hand by so-called frequentists (who assert that probability exists in nature), Bayesians (who hold that all uncertainty reflects imperfect knowledge or a simplified model of the universe), with a middle ground of some sort represented by Der Kiureghian and Ditlevsen.

As a test of the three viewpoints, consider a horse race. It takes place on a particular day and time, with particular weather and track conditions and with horses and jockeys in an unrepeatable mental and physical state. Is the outcome of the race aleatory or epistemic? I assert that, unbeknownst to all, one horse and jockey are the fastest pair under these conditions, and *will* win. But the experiment will only be held once, never repeated. Does the probability distribution of the winning horse exist in nature (frequentist), does uncertainty about the outcome solely reflect one's knowledge state and model of the universe (Bayesian), or does it depend on whether the person making the bet is in a position to gather knowledge from the feed room (Der Kiureghian and Ditlevsen)? If the quantity of interest can only be observed once, with no possible repetition to estimate the frequency with which each horse will win, does its probability distribution exist in nature, or is it reducible with better knowledge? Both definitions employed by frequentists seem to break down in this example, the Bayesian viewpoint holds up, and Der Kiureghian and Ditlevsen's definition cannot be applied without more knowledge about who the bettor is.

Figure 4. Suffolk Downs starting gate during a live horse race, from August 1, 2007. Can the probability mass function of its outcome be said to exist in nature? (Image credit: Anthony92931, Creative Commons Attribution-Share Alike 3.0 Unported license)

Why I hold the Bayesian viewpoint

What is the value in calling an uncertainty "aleatory" if aleatory does not mean what it is supposed to mean, if it does not mean irreducible, if one cannot be sure the uncertainty exists in nature? Words are only useful in technical writing if they mean what we want them to mean. I suggest that writers who do not believe that aleatory means what they want it to mean should not use the word, regardless of what other people think.

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